A Pluralist Approach to Proof in Mathematics

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      The term 'pluralism' has been used by philosophers of mathematics to denote an attitude, amongst others, in their philosophies. Philosophers of mathematics who list pluralism as a virtue of their philosophical position include Shapiro and Maddy.[1] Philosophers engaged, not so much in whole philosophical system building, but in analysing particular aspects of mathematics from a philosophical point of view are usually Pluralist, at least in some respects. The Pluralist is inspired by Shapiro's structuralism, Maddy's naturalism, the observed behaviour of mathematicians and a number of remarks made by mathematicians and logicians concerning the phenomenology, heuristics and the role of proof in mathematics. As it is being used here, the term 'pluralism' is closely allied to the term 'formalism' as it is used by mathematicians (and not philosophers of mathematics).

      Pluralism is now being developed as a philosophical position.[2] The Pluralist philosopher of mathematics, is someone who is tolerant of different orientations in mathematics, of different foundations (which conflict in what they say about the essence of mathematics),[3] of conflicting truths in mathematics, since truth is always relative to a particular theory.[4] In this paper, I shall focus on what the Pluralist has to say about proof in mathematics. We'll begin by recounting the Pluralist take on Maddy's version of Naturalism,[5] which motivates the philosopher to take seriously the behaviour and avowals of working mathematicians in developing a philosophy of mathematics. In the following section, we'll make some observations about published proofs and look at some mathematician's avowals concerning proofs. In the final section, we'll state how it is that the Pluralist views mathematical proof.

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[1] There are not many others. Most philosophers of mathematics are foundationalist, which makes them either monists or dualists. See Michele Friend "Pluralism and "Bad" Mathematical Theories". Presented at the World Congress on Paraconsistency, (Melbourne July 2008).

[2] Michele Friend "Pluralism and "Bad" Mathematical Theories". Presented at the World Congress on Paraconsistency, (Melbourne July 2008). Forthcoming. Michele Friend "Meinongian Structuralism" Proceedings for Logica ????

[3] The Pluralist is anti-foundationalist in the traditional sense, but, at a very general, abstract level of discussion, does adopt a logical foundation, he is not fixed on that logic. Alternatives are also possible. This ensures that the pluralist is pluralist about his pluralism. "I smell the waft of contradiction" I hear you say. I reply: there are several logical systems now which can cope with contradictions. None is privileged *tout court*.

[4] This idea is of structuralist inspiration. See Stewart Shapiro Structure and Ontology ????

[5] Michele Friend "Some Problems with Naturalism" Presented at the Association of Symbolic Logic European Summer Meeting, Sofia, August 2009.