

Philosophy 196.10**Topics in the Theory of Knowledge:**
Philosophy of Mathematics**Spring Semester: January - May 2008****Class time:** Tuesdays and Thursdays 12:45 – 2:00**Room:** 309 in 1957 E street.**Professor:** Michele Friend**Office:** 309 Academic Building, Mount Vernon Campus.**Office hours:** 5:30 – 6:30 Tuesday, or by appointment.OR: **Office:** Foggy Bottom Campus, Phillips Hall T510B.**Office hours:** 2:15 – 3:15 Tuesdays or by appointment.**Office telephone:** (202) 242-6708. I do **not** reliably check the voice mail.**e-mail:** michele@gwu.edu.**Texts:** Friend: Introducing Philosophy of Mathematics

Shapiro: Philosophy of Mathematics; Structure and Ontology

Articles will be available in the library for photocopying.

Grading: Assignment I:	15 %
Assignment II:	50 %
Assignment III	15%
In class test:	20 %
Total:	100% There is no final examination.

Grading policies and attendance:

I am not responsible for ‘getting you through the course’, or ensuring that you ‘get an ‘A’’. That responsibility falls on you. I am here to assess your performance and understanding of the subject matter covered in class. Your understanding might go beyond what is discussed in class. For greater understanding you need to do some independent thinking, or some research. Consult the bibliographies of the texts. To pass the course, you need 45 %. Letter grades are distributed as follows: 95 – 100: A, 90 – 94.5: A-, 85 – 89.5: B+, 80 – 84.5: B, 75 – 79.5: B-, 70 – 74.5: C+, 65 – 69.5: C, 60 – 64.5: C-, 55 – 59.5: D+, 50 – 54.5: D, 45 – 49.5: D-, 0 – 44.5: F.

Assignments are due in class, or can be handed in **before** the relevant class. If you are handing an assignment in early, then please ask the secretary in the Philosophy department to put it in my mail box. Late assignments are not accepted. No grade is given in the case of a late assignment or an assignment not handed in. You simply lose the grades. You will not have a chance to re-do or make-up an assignment. Get it right the first time.

‘Assignments’ and ‘homework’ are not the same thing. Assignments are handed in, given a grade and count towards your final grade. Homework exercises are set almost

every class. These are practice exercises for you. Standardly, they are not handed in. They are not graded. If you wish to have your homework checked, or if you do other exercises, then you may hand them in to me or to my mail box in the secretary's office in the Philosophy Department. The homework will be checked by me. I shall simply indicate any problems. I'll return 'corrected' homework in class.

It might be useful to form study groups, where you help each other with the homework and assignments. Statistically, students do better in the class, when they form study groups. However, be careful that study meetings do not degenerate into sessions where one person does the work, and the others copy. If you form a study group, make sure that all the people in the group understand the solutions. If you do one of your official graded assignments with someone else, please write this at the top of the page. This is for reasons of academic integrity. No marks are deducted for working in groups. **You should all hand in your own copies of assignments. This means more than just printing out the 'solutions' and putting your own name at the top. You should each write out the solutions separately in your own words.**

Again, statistically, it is indicated that attendance in class is important. Attendance is not compulsory. The responsibility is entirely yours for catching up on missed classes. It is a good idea to ask a fellow student if you can copy his, or her, notes and discuss the content of the lecture you missed. In the great majority of lectures, new material is covered, and the knowledge is cumulative and cross-referring. That is, you will find it hard to progress if you neglect a part of the course. If you come ill prepared to class, you risk being marked as "absent"! You should be able to show that you have at least attempted the homework.

It quite often happens that at some point a student will feel that he, or she, is falling behind, or feels that he, or she, does not understand. It is also not always easy to assess whether or not you do really understand. A "warm feeling" is not equivalent to understanding. You have to be able to display your understanding. If you realize that you do not fully understand the material, it is a **bad** idea to avoid doing the work, coming to class, and coming to see me or the logic tutor during office hours. It is a **good** idea to see me, come to class, ask your friends in the class to help you and continue to hand in assignments, however sketchy.

The logic tutor is someone you can visit and ask for help. Asking me, or the logic tutor, for help will not count against you in any way. If you have kept up with the work, and resolved problems as they arose, then your study for the final exam should be minimal!

The prerequisite for the course is introduction to logic. It is a good idea to have also taken the course in Symbolic Logic, or something equivalent. If you have any doubts about this, please see me after class.

Course description:

In general, the philosophy of mathematics differs from the philosophy of other areas of enquiry because of the presuppositions which are made concerning the nature of mathematics. The philosophy of mathematics differs from the philosophy of science

ontologically in that a lot of (arguably all of) mathematics cannot be scientifically tested, in the sense of using observations and laboratory experiments to decide whether a proposition in, or about, mathematics is true. This is because a lot of mathematics concerns abstract objects. Moreover, a lot of mathematics concerns infinite objects, or infinite collections of objects, and we cannot physically observe or test these. The philosophy of mathematics differs from other areas of philosophy epistemologically. This is because our methods of enquiry in mathematics are deductive, in the sense of requiring rigorous proof. There is no sense in which the claims of mathematics are decided upon by vote, consensus, coercion, through political mechanisms or with respect to social or moral concerns. The philosophy of mathematics differs from some other areas of philosophy phenomenologically, in that mathematics is “cold”. We do not (usually) care very much whether the answer to a mathematical problem is one number or another. *Prima facie*, this all seems right. We shall revisit these presuppositions in the class.

The basic questions we ask in the philosophy of mathematics concern ontology and epistemology. Ontological questions we might ask include: what is it that we study when we study mathematics? Are numbers independent of us? How many of them are there? What make an object abstract? Is an abstract object less real than a physical object? Epistemological questions we might ask include: How do we choose between one area of mathematics and another? Are some formal proofs better than others? Are we discovering or creating mathematics? What is (are) the mechanism(s) which allow(s) us to understand mathematical truths?

In the course, we shall start with an investigation into infinity. The text we shall use is: *Introducing Philosophy of Mathematics*. We start with infinity because almost every philosophy of mathematics has something important to say about infinity. Furthermore, it is not a bad idea for students to have some understanding of how it is that mathematics deals with infinity. We shall then examine the default, or naïve, philosophical position called Platonism. Of course, Platonism has grave problems associated with it, so we shall then look at an amendment to Platonism, called Logicism. The third position we shall examine is Structuralism, which can usefully be compared to Platonism. We shall then plunge into a more advanced text which sets out the structuralist position in detail. The text is by Shapiro. The title is: *Philosophy of Mathematics; Structure and Ontology*. This book is a very carefully worked out presentation of the structuralist position, and will add depth to our understanding of the philosophy of mathematics, since we shall see some debates carried out at a sophisticated level.

Academic integrity:

In 1995, the students of George Washington University committed themselves to high standards of academic integrity by passing a 'Code of Academic Integrity'. By attending George Washington University you are obliged to follow this code. The 'Code of Academic Integrity' can be found in the "Student Planner and Handbook" and also in the "Guide to Student Rights and Responsibilities". The latter can be obtained in the office of the Dean of each school, and in the office of the Dean of Students. You are expected to be familiar with the Code, to understand what constitutes dishonest academic behavior, and to avoid any such behavior. Sanctions for offences include failure of an assignment, failure of a course, suspension from the university for a specified amount of time and expulsion from the university.

Instructional Modifications for Students with Disabilities:

Should any student require any modification to the testing procedure, the seating arrangement or access to the classroom please speak to me either after class, or during office hours. Such matters have to be dealt with by an official procedure, for the most part. So, it is better to set this in motion as soon as possible.

Schedule: The schedule is a rough schedule. If the University stops operating, because of snow or terrorist activities, then the schedule just gets bumped up a session. That is, the dates might change, but not the order or content of lectures. A make-up day for each day missed will be announced towards the end of the semester. Notice will be given in class of any changes. Homework is due in the next class. Assignments are due two classes after they are assigned. So, usually, assignments are due a week after they are assigned. For the homework topics, write about a page answering the question. How to read the schedule: work from left to right. We start with the date, then we have the reading for that day's lecture. We then have the lecture, followed by the homework assigned that night.

Date	Reading for class	Topic covered in class	Homework/ Assignments.
Tues. 15 Jan.	Friend 1 – 7.	Syllabus, infinity: Zeno's paradoxes.	In your own words explain one of Zeno's paradoxes. Is there a good solution to it? (If so, what is it?).
Thurs.17 Jan.	Friend 7 – 12.	Potential versus actual infinity.	Compare and contrast potential infinity with actual infinity.
Tues. 22 Jan.	Friend 12 – 13.	The ordinal notion of infinity.	What is the difference between a natural number, an ordinal number and a cardinal number?
Thurs. 24 Jan.	Friend 13 – 22	The cardinal notion of infinity.	Re-write out Cantor's diagonal proof in your own words. Is it really a proof?
Tues. 29 Jan.	Friend 23 – 26, Plato: The Meno	Platonism, The Meno.	Is Plato right about mathematics?
Thurs. 31 Jan.	Friend 26 – 47.	Modern Realism in mathematics	What are the main differences between Platonism and Realism?
Tues. 5 Feb.	Friend 49 - 63	Fregean Logicism	What is Fregean logicism? What if Frege reacting to?
Thurs. 7 Feb.	Friend 63 – 79	Russellian Logicism	Assignment I set. Due 14 Feb. Handed out in class.

Tues. 12 Feb.	Friend 81 – 85, Benacerraf: “What Numbers Could Not Be”	Introduction to Structuralism. The motivating puzzle.	None.
Thurs. 14 Feb.	Friend 85 - 100	Structuralism: Hellman, Resnik and Shapiro	Distinguish the three structuralist positions.
Tues. 19 Feb.	Shapiro 21 – 44	Preliminaries to Shapiro’s structuralism	None.
Thurs. 21 Feb.	Shapiro 44 - 67	More preliminaries to Shapiro’s structuralism	Distinguish between realism in ontology and realism in truth- value. Discuss Shapiro’s realist commitments. Propose a topic for assignment II, set up an appointment with Friend to discuss it. You may hand in drafts, or partial drafts during the next few weeks.
Tues. 26 Feb.	Shapiro 71 – 90	Structuralist ontology	What is the difference between more traditional forms of realism and structuralist realism.
Thurs. 28 Feb.	Shapiro 90 – 106 Potter and Sullivan "Hale on Caesar", Philosophia Mathematica, 5 (1997), 135—52.	More on the ontology	Is Shapiro’s ontological commitment plausible? The Potter and Sullivan paper does not directly address Hale, but it does have repercussions for Shapiro.
Tues. 4 Mar.	Shapiro 109 - 120	Epistemology and infinity	According to Shapiro, how do we know about infinite totalities?
Thurs. 6 Mar.	Shapiro 120 - 143	More on epistemology	Are Potter’s criticisms relevant to Shapiro’s structuralism? Why, why not? Is Shapiro in trouble?
Tues. 11 Mar.	Potter: "What is wrong with abstraction?", Philosophia Mathematica, 13 (2005)	A critique of abstraction.	What is abstraction, what has it to do with epistemology? Is it necessary for mathematics?

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Thurs. 13 Mar.	Shapiro 143 - 180	Revisiting structuralism. General discussion.	Study.
Tues. 25 Mar.	None.	Test	None.
Thurs. 27 Mar.	Shapiro 181 - 189	Construction, modality logic.	Assignment II formally “set”. Due Thurs. 3 April.
Tues. 1 Apr.	Shapiro 189 - 216	More on construction, modality, logic.	None.
Thurs. 3 Apr.	Shapiro 216 - 228	Modality, structure, ontology	None.
Mon. 7 Apr.	None	Thacher Lecture by Shapiro. Time and place will be announced	None
Thurs. 10 Apr.	Shapiro 228 - 243	More on modality, structure and ontology	None
Tues. 15 Apr.	Shapiro 243 - 263	Structures and reality	Half of class prep 5 min. synopsis on a position you would defend regarding structuralism.
Thurs. 17 Apr.	None.	General discussion	None.
Tues. 22 Apr.	Friend 101 - 114	Constructivism	Half of class prep 5 min. synopsis on a position you would defend regarding constructivism or infinity.
Thurs. 24 Apr.	Friend 114 - 124	More on Constructivism. General discussion.	Assignment III handed out in class. This is due 1 May. Please put it in my mail box in Philips 525. Put your name on it, and my name on it!

There is no final examination.

After the class: The grades should be in to the registrar’s office by 9 May. I am not allowed to divulge the grades to you by e-mail until the registrar has published the final grade. You may pick up your graded assignment III from the office of the secretary of the philosophy department, once these have been graded. If you would like to learn more from your assignment, then you may come to see me in the following fall semester. If you would like to continue studying the philosophy of mathematics, then there is an informal seminar taught by Friend in the fall semester under the auspices of Readings and Research. You have to be signed in to the class, contact me just before fall semester to find out about the meeting for the first class which will be largely organizational. You’ll be signed in then. This class is a pre-requisite for the Readings and Research seminar. In

this seminar we look at a top research book in the philosophy of mathematics in depth. There are also some appropriate classes in the department of mathematics, and at some other universities in Washington D.C.

Future Logic Tutors: This is a note to those of you who enjoyed doing the formal proofs in logic, and who feel that you would be interested in the logic tutor position in the department of philosophy. This is how it works: before the Christmas break in the fall semester, submit a short letter of application together with a copy of your transcript to the mail box of Michele Friend in Phillips 525. Some of you will be interviewed in the early Spring of the following year. Should you be chosen as the next logic tutor, you will be asked to tutor for 1.5 hours per week for the department. Half of one of those hours, should overlap with the established tutor, so that he, or she, can impart some wisdom to you. In the following Fall semester, you would be the sole logic tutor. You would work for 3 hours per week for the philosophy department. In the following Spring semester, you would then teach for 1.5 hours per week, overlapping with the future logic tutor.

Other texts to read: See the bibliography of Friend: *Introducing Philosophy of Mathematics*.